Cayley-Dickson Trans-Sedenion Hyper-Complex C*ndyl*nd

The Rules of the Game

The first time you play, cut 32 little cards from the paper provided. Each card has a letter or number on the face and some equations on the back.

Each player provides a coin for their token. It is important whether the coins are Heads Up or Tails Up, so be careful not to flip them unintentionally.

The game board contains 32 cells, either circles or squares. The first line printed in each cell is its name. The remaining lines are a table of moves. The white square named "1" is also marked "START", and the burgundy circle named "ABCDE" is also marked "FINISH".

To start, all players place their coins Heads Up on the cell marked "START".
Shuffle & cut the deck, and deal 3 cards face down to each player for their initial hand.
Place the rest of the deck face-down for drawing.

Players take turns drawing 1 card and adding it to their hand, and then picking a card from their hand to play. When a card is played, it is placed face up on a discard pile. After drawing the last card, shuffle the discard pile and turn it face down for more drawing.

When you play a card you also move your coin and possibly flip it. If the card has a letter (a, b, c, x, or y), look up that letter in the multiplication table in your current cell, and move your coin to the cell it indicates. If the destination cell is preceded by a minus sign (-), also flip your coin over (from heads to tails, or from tails to heads up). If the card has a number 1 or -1, your coin stays in the same cell when you play it. If it is -1, you flip the coin over.

For instance, if you are on cell "BXY" and you play an "a" card, the "BXY" cell says

$$a \rightarrow \text{-abxy}$$

so you move your coin to cell "ABXY" and flip your coin over. (The instructions in the end-zone of the game board give an example with a two-letter card "ax", but multi-letter cards are not in the basic card pack. See the advanced card pack for those! Other than that, the examples in the end-zone correctly show you how to move.)

The Pauli Exclusion Principle dictates that no two coins may be on the same cell in the same orientation. If you move to a cell where another player already has a coin with the same side up as your coin, you shout "Fermi-Dirac Statistics!" and you send them back to START, without flipping their coin. This applies to all cells except START (where coins are bosonic).

The goal is to end your turn with your coin Heads Up in the cell marked FINISH. The first player to do so wins. If it's in the FINISH cell but Tails Up, it's not finished yet -- keep on playing!

Glossary and Notes

Trans-Sedenion: The names on the cells and the values on the cards represent the 32 unit bases of the Trigintaduonion Numbers. These are the sixth in an infinite sequence of number systems that goes **Real Numbers, Complex Numbers, Quaternions, Octonions, Sedenions, Trigintaduonions, ...,** and each one includes all of the previous ones as subgroups. Our game uses the first trans-sedenion number system, which has 1 "real" basis (the number 1) and 31 "imaginary" bases (which have alphabetic names on the board: a, b, ab, c, ac, bc, x, ax, bx, ..., abcxy). They are called "imaginary" because if you multiply any of them by themselves you get -1, and no "real" number can do that. Our "a" is what mathematicians usually call "i" in the Complex Numbers, and our "a", "b", & "ab" are what Hamilton called "i", "j", & "k" in the Quaternions.

Cayley-Dickson: The most popular way to construct these hyper-complex numbers is the Cayley-Dickson construction, which is written on the backs of the cards. It tells us exactly how to multiply our hyper-complex numbers, if you break them into halves, each half being of the previous number system in the sequence. (*P* and *q* are the halves of one number; *r* and *s* are the halves of the other.)

Candyland: A trademark of Hasbro for a game that is infamous for boring anyone except little kids because there is no strategy involved in playing it. Our game contains 0% real candyland.

Heads or Tails Up represents the sign of your position, positive or negative. The Trigintaduonion basis units are closed under multiplication if you include their negatives, so there are actually 64 numbers in play.

When you make a move, what you are actually doing is multiplying the number of your position (including its positive or negative sign) times the number on the card, and moving your coin to the product. You can check that "1" times any letter is that letter (because "1" is the multiplicative identity element), and that any single-letter cell times itself is "-1" (because they are imaginary units). Multiplying Trigintaduonions is a little weird because neither the Communtative nor the Associative identity applies! But they do have some other properties that you may discover or read about.

Fano Planes: The four shaded triangles on which the cells are laid represent a diagram called the Fano Plane, which is a guide for how Octonion unit bases are multiplied. If you ignore "x" and "y" in our game, you get Octonion numbers with primary bases named "a", "b", & "c".

Can of Worms: The shields in the end-zone are iconic of these number systems: Real Numbers are for measuring distance, and Complex Numbers are great for rotations in 2D or periodic sine waves as in AC electronics. When William Rowan Hamilton discovered the Quaternions in 1843, he was so excited he wrote their equations on a bridge in Dublin, Ireland. The Octonions are represented by the Fano Plane. The Sedenions and and Trans-Sedenions have an unfortunate feature called "zero divisors": it is possible to multiply a number by zero and not get zero! For many mathematicians this is a deal-breaker, so it opens a can of worms!

Learn More by searching the internet for the terms above, and find more information, references, and the Advanced Card Pack on the web: `http://wiki.yak.net/1093`

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